BOSCOVICH'S "MODEL OF ATOM" FROM 1748

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Boscovich's theorem on the equilibrium state of three points and its generalizations, which in their basic conception are extremely close to Bohr's model of atom from 1913, previously were exclusively explored in the form in which Boscovich incorporated them in his famous work *Theoria philosophiae naturalis*. (1) Boscovich's results will be considered and evaluated here at the point of their commencement in 1748, ten years before the publication of the first edition of *Theoria*, that is, within the framework of the genesis of Boscovich's curve of forces (*curva Boscovichiana*).

Theorem on the Equilibrium State of Three Points

In the second part of his treatise *De lumine* (1748) Boscovich concluded the shaping of his curve of forces with an essential development in relation to his original exposition in *De viribus vivis* (1745). That is to say that Boscovich made concrete the meaning of null points of his curve of forces when he interpreted the cohesion of matter (Fig.1). (2) Let a particle of matter lay in the origin of coordinate system A, a second particle in one of the null points of curve in which the repulsive force turns into an attractive force (under the condition that the distance between these points is increasing), for example in the point H. Should the particle be removed from point H as a result of action of external force so that the distance between the particles is either increased or diminished, then the force that tends to

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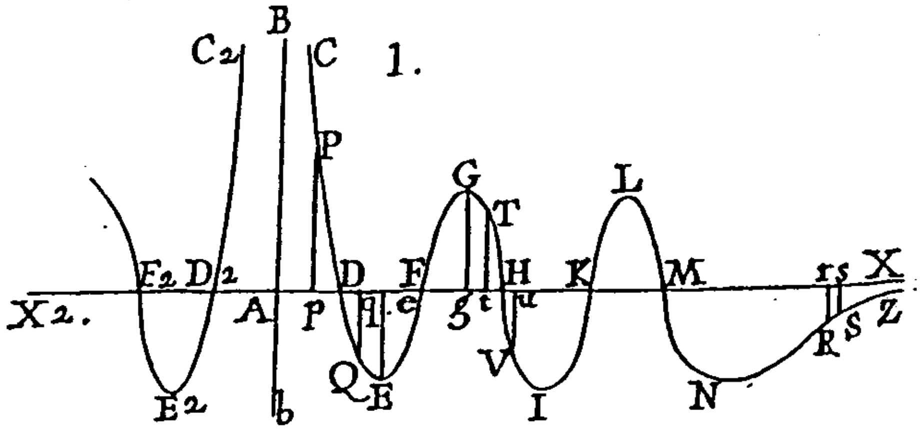
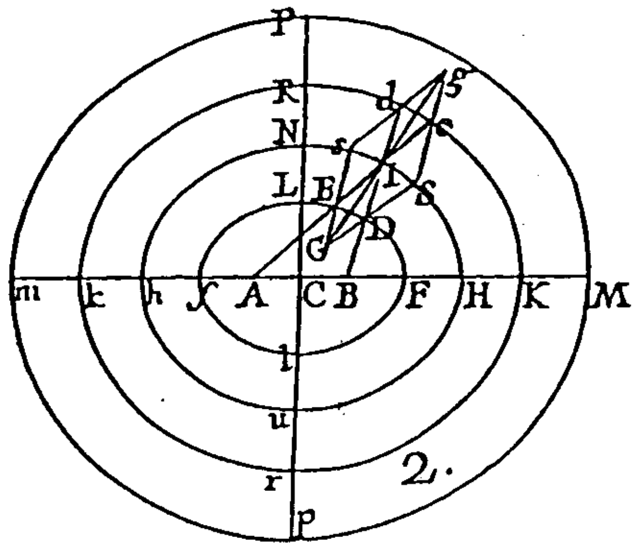
return the particle to the initial point H should appear. It is therefore quite justifiable to name such a null point of curve as the limit of cohesion. The opposite case occurs when the second particle is placed in the null point of curve in which the attractive force turns into repulsive force under the condition that the distance between these points is increasing, for example in the point F. Should the particle be removed from point F as a result of action of external force so that the distance between the particles is either increased or diminished, in either case the force that tends to distance the particle from the initial position F should appear. Such a null point is justifiably called the limit of noncohesion. With the distinction of the limits of cohesion and noncohesion Boscovich dynamically interpreted the null points of his curve of forces.

Immediately after the interpretation of cohesion of matter Boscovich interpreted its solidity. As a model for the research of this physical quality of matter he chose the simplest case, which for him meant the case with the most elegant consequences: system of three points that do not lie on the same straight line. (3) First of all he introduced the correspondence between the forces-ordinates of his curve of forces and the forces-vectors that act in the direction of distance between points that are distributed in plane. Boscovich's choice induced the mathematical tools appropriate for the application of Boscovich's law of forces to the spatial distribution of points: vector analysis and the theory of conic sections. In such a way with the addition of vectors by the method of parallelogram he obtained the composite force by which two points of system jointly act on the third. Already at the beginning of his research he defined the following ellipse (Fig.2): he placed the two points of the system, in wich the distance is equal to the interval between the two limits of cohesion, in the foci of ellipse A and B; for the transversal semiaxis he chose the distance from the origin to the limit of cohesion AH; and the third point I of the system he placed on the perimeter of ellipse. The characteristic of radius-vectors

$$AI - CH = CH - BI$$

corresponds to the situation on figure 1, that is, the abscissae Au, At differ equally from AH. This justifies Boscovich's additional supposition about the flux of the curve of forces in the neighborhood of the limit of cohesion H: arcs HT, HV are quite similar and equal, that is, in the points of abscissa that are equally distant from H there appear forces of e-





Figures 1 and 2. From Boscovich's treatise <u>Dissertationis de lumine pars secunda</u>, (Roma, 1748). Courtesy of the Historical Archives (Historijski archiv), Dubrovnik.

qual intensity and contrary direction. Only after he included that supposition Boscovich provided with the methodology for the invention of equilibrium positions of point I on the perimeter of ellipse. If I is placed in the vertex H of transversal axis then I is in equilibrium because in that position the attractive force destroys the repulsive force. If I is placed in the vertex N of conjugate axis then I is again in equilibrium because by definition any ellipse equals.

AN = BN = CH (Fig.2) = AH (Fig.1)

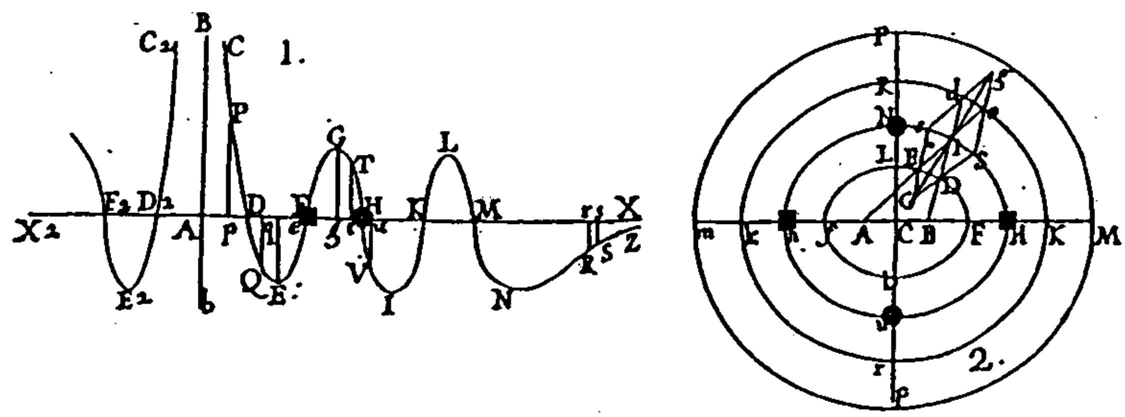
and at the limit of cohesion H both attractive and repulsive force disappear. Should, however, point I be somewhere on the perimeter outside vertices, with the help of established methodology it is easy to conclude that two opposite forces (one attractive and the other repulsive) act on the particle. These forces stretch parallelogram whose diagonal has the direction of tangent on the ellipse toward the proximate vertex of conjugate axis. On the basis of these considerations Boscovich formulated his theorem: "Point placed in any vertex of any axis will be on the limit of attraction and repulsion. But placed anywhere on the perimeter of its ellipse it will possess the force which moves the same point in the direction of the same perimeter toward the most proximate vertex of conjugate axis". (4)

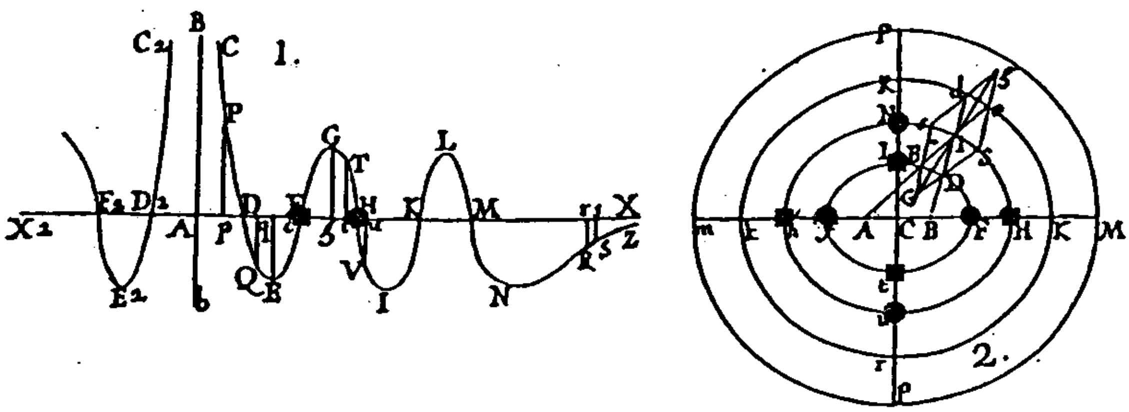
The Power of Analogy

From that point Boscovich turned toward the exploration of analogical thought. Specifically, in several steps he deepened the correspondence between the null points of his curve of forces and equilibrium states of the third point in the system he explored. From the letter of the theorem it follows directly that the behavior of the third point of the system in the neighborhood of vertices N, n (Fig.2) of conjugate axis is analogous to the behavior of the particle in the neighborhood of null points H, M (Fig.1), wich he even called the limits of cohesion. The behavior of the same point of the system in the neighborhood of vertices H, h (Fig.2) of transversal axis, too, is analogous to the behavior of the particle in the neighborhood of null points F, K (Fig.1), which he called the limits of noncohesion. Therefore, with the same explanation the vertices of conjugate axis must be called the limits of cohesion and the vertices of transversal axis must be called the limits of noncohesion. The analogy does not refer only to the dual character of limits, but also to their vicissitude: on the perimeter of ellipse the limits of cohesion and noncohesion alternately follow each other. This was Boscovich's first analogy (Fig.3).

Boscovich discovered the second analogy by observing the system of confocal ellipse in which the transversal semiaxes were alternately equal distances AF, AH, AK, AM (Fig.1) from the limits of cohesion and noncohesion to the origin of coordinate system. Ellipse generated by the distance AH between the limit of cohesion and origin has, as was already noted, the limits of cohesion in the vertices of conjugate axis and has the limits of noncohesion in the vertices of transversal axis. To the contrary, ellipse generated by the distance AF between the limit of noncohesion and origin has, as can be determined with the help of established methodology, the limits of cohesion in the vertices of transversal axis and has the limits of noncohesion in the vertices of conjugate axis. As two types of limits that alternately follow were established in Boscovich's curve of forces, so, too, two types of confocal ellipses with alternative distribution of limits of cohesion and noncohesion in its vertices were now discovered by analogy (Fig.4).

The alternate series of limits of cohesion and noncohesion was the fundamental idea of Boscovich's third analogy. Boscovich again considered the ellipse in which the transversal semiaxis was equal to the distance AH between the origin and the limit of cohesion of his curve of forces and then proved with the same methodology that the third point I of the system, exiguously distanced from the perimeter, will tend to approach that perimeter if it is placed within or outside the perimeter. Therefore, the whole perimeter of such ellipse behaves as a limit of cohesion sui generis. In the same way, concerning the ellipse generated by the distance AF between the origin and the limit of noncohesion he proved that its whole perimeter behaves as a limit of noncohesion because the third point I of system, which is exiguously distanced from such perimeter, will always tend to run away from it. That means that the very system of confocal ellipses represents one <u>new</u> series of limits of cohesion and noncohesion (Fig.5). The motion of the third point is there completely determined. If the point I is placed on any perimeter it tends to move along that orbit. If, however, it is placed between two perimeters, then it distances itself from the one that figures as a limit of noncohesion and approaches the one that figures as a limit of cohesion. This <u>planimetric</u> conception is included in Boscovich's <u>Theoria</u>, so that the previous researches that were based on the study of





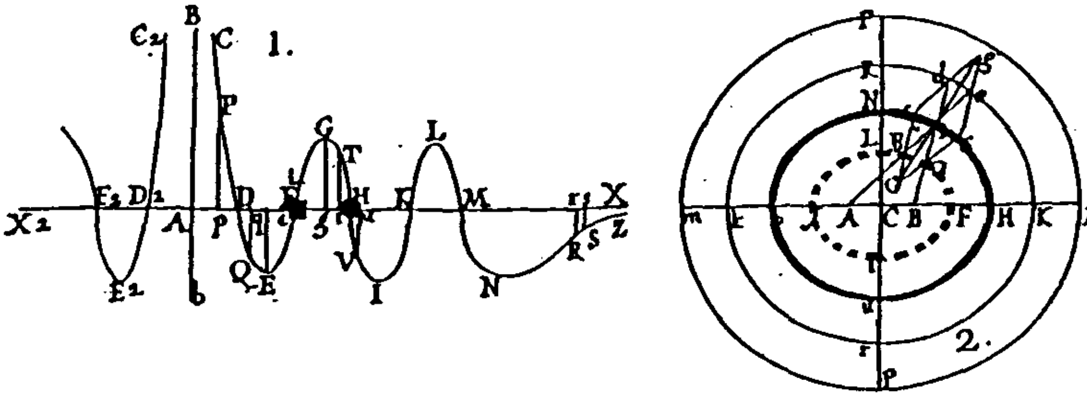
●— limit of cohesion

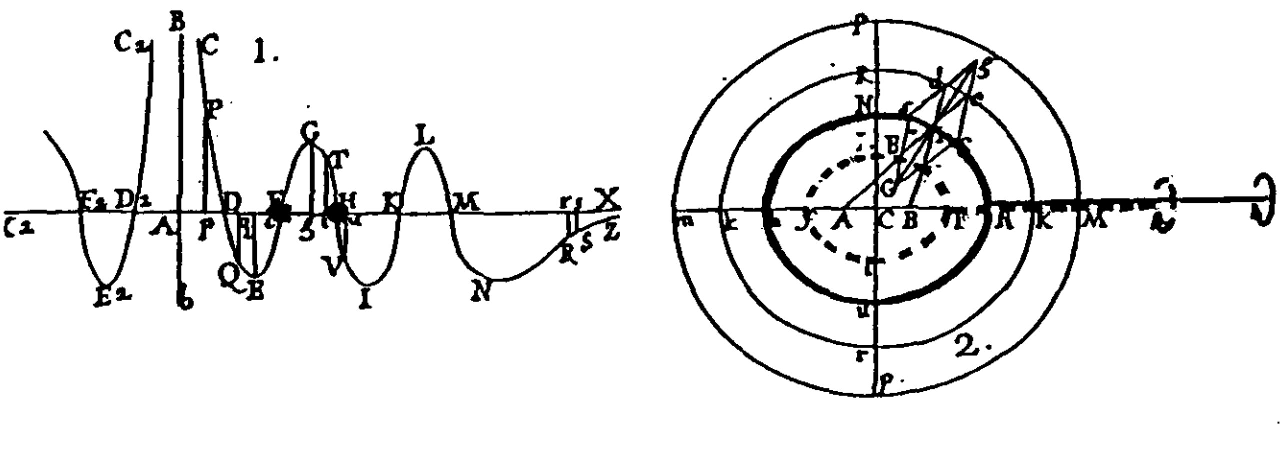
■-- limit of non-cohesion

Boscovich's four analogies with the limits of cohesion and non-cohesion

Figure 3 - Vertices of en ellipse.

Figure 4 - Vertices in the system of confocal ellipses.





● — limit of cohesion

■ - - limit of non-cohesion

Figure 5 - Whole perimeters in the system of confocal ellipses.

Figure 6 - Whole surfaces in the system of confocal spheroids.

Boscovich's Theoria regularly pointed to this finding of Boscovich as a brilliant anticipation of Bohr's model of atom.

Boscovich, however, went one point further in 1748: "What was said about the perimeters holds also for the surfaces of sphereoids generated by the revolution around transversal axes". (5) Boscovich, therefore, conceived the system of confocal spheroidal shells that with their whole surfaces represented the limits or equilibrium states of the third point of the system (Fig.6). The particle that moves along such a surface will spontaneously continue to move on this surface. The particle that would arrive in the space between two spheroidal shells would move toward that shell that corresponds with the limit of cohesion and would be distanced from that shell that corresponds with the limit of noncohesion. This was the spatial conception which Boscovich never again repeated, enriched, or applied; not even in Theoria. The speculative models of atom from the beginning of the twenthieth century, (6) which took shape before the experimental verification of E. Rutherford, for example J. Perrin's structure nucleoplanetaire (1901), H. Nagaoka's Saturnian system (1904) and J. J. Thomson's allowed and forbidden orbits of corpuscles (1907), had the exactly based forerunner in Boscovich's spatial "model of atom". Moreover, in the case of J. J. Thomson one can detect a direct inspiration in Boscovich, although Thomson knew only Boscovich's planimetric conception.

Boscovich's Four "Not to Be Omitted"

The system of confocal spheroidal surfaces as a spatial conception for equilibrium states of the system of three points was not the highest achievement of Boscovich's imagination. According to Boscovich himself "numberless other matters remained that would be worthy of mention". (7) Nevertheless, Boscovich concluded the discussion on the system of the three points with the four claims that "should not be omitted". (8) In that way he delineated four new directions of research.

In the first place Boscovich questioned the additional supposition about the flow of his curve of forces in the neighborhood of the limit of cohesion or noncohesion. If the arcs HT, HV on Fig.1 are not equal, then, with the help of established methodology, it is no longer possible to obtain ellipses but some other curves that can be determined by the inverse method of tangents. If the forces-ordinates in the neighborhood of the point H on Fig.1 are given, then arc of orbit can be found by this method as a new equilibrium state of the third point of system. And vice versa, if such an arc is given, then a flow of the curve of forces in the neighborhood of the point H can be determined. This idea of Boscovich from 1748 offered a new possibility for the interpretation of different physical phenomena in the light of his theory of forces. Boscovich obviously expected great results from it, expecting at the very least the methodological exercises in analysis and geometry. But in 1758, when in his Theoria Boscovich systematized his thirteen-year long investigations in the theory of forces, he publicly gave up searching for the curves that in the interpretation of some physical phenomena would play the role analogous to the role played by the ellipse in the interpretation of equilibrium state of the system of three points: "But I am going to omit all discussion of that kind, because it does not seem to me appropriate for the application of [my] Theory". (9)

Boscovich's second idea refers to the centrifugal force as a new dynamic component for the determination of equilibrium state of three points. This idea is basically the idea of total dynamic balance of the system, which Boscovich never again accented so distinctly. If the centrifugal force is equal to the excess of the attractive force of the farther point over the repulsive force of the nearer point then the third point of the system will oscillate around the vertex of conjugate axis or circulate along the whole perimeter, depending on the initial impulse. The perimeters of confocal ellipses, or rather the surfaces of confocal spheroids, are here already stationary states from Bohr's first postulate. Neverthless, there is no energetic argumentation of the transition from one stationary state to another. This argumentation is a content of Bohr's second postulate according to the quantum theory

$$h\nu = E(n_2) - E(n_1)$$

Boscovich's third idea reminds us that the supposition of the stillness of points A and B placed in the foci of ellipse is an essential simplification in the research of the equilibrium state of the system. And these points can be and are subjected to the action of attractive and repulsive forces, so that this influences the movement of the third point. The additional impediments must be included in the considerations of the system.

Boscovich's fourth attitude investigated how the third point of the system would move if it came from infinity, namely if the angle between radius-vectors AI, BI disappeared. Carried away by the wish to consider the case of three points in an all-inclusive way, Boscovich here apparently forgot that on his curve of forces he did not mark the limits of cohesion and noncohesion in the infinite distance from the origin of coordinate system. Or, despite this, was he considering this coming from infinity as an important case?

The Historical Context of Boscovich's Idea

Having started from the continuous curve of forces and the discrete distribution of three points Boscovich arrived at his original interpretation of solidity of matter and thereby in a new area confirmed the fundamental duality of his theoretical physics, namely, the duality between continuous force and discrete matter. This new relationship between continuous forces and discrete points of matter was that elegant consequence which Boscovich proclaimed as the new name for simplicity in geometrical research. Had it not been so, in the original fragment from De lumine Boscovich would had studied a trivial case of three points which lie on the same straight line, exactly as he started his systematic exposition in Theoria. (10) This would not have taken him very far.

In order to research the equilibrium state of three points, and in order, finally, to establish the new spatial conception of the system of three points, Boscovich chose the theory of conic sections as the appropriate mathematical tool. This choice was conditioned by the fact that a year earlier, that is, in 1747, Boscovich's Elementa sectionum conicarum were "in greatest part prepared and anticipated only the final brush-up", (11) although this tome of Elementorum universae matheseos was published only in 1754. At the same time this is an example of how mathematical research often enriched physics, this aspect being an insufficiently noted characteristic of Boscovich's creative work.

The shaping of qualitative flux of Boscovich's curve of forces decisively influenced the genesis of the theorem on the equilibrium state of the system of three points from 1748. At the same time, this theorem went before the formulation of six mathematical conditions that must satisfy the curve of forces, as Boscovich himself carried out in his later treatise De lege virium in natura existentium (1755). (12) Therefore, this theorem belongs to the early developmental stage of qualitative description of Boscovich's curve of forces and was a forerunner to Boscovich's unsuccessful attempts at exact analytical expression of that curve. Immediately before the formulation of the theorem Boscovich introduced the limits of cohesion and noncohesion as a conceptual novum in his theory of forces. He recognized these same limits in the research on the conditions for the equilibrium state of the system of three points. Moreover, having aspired toward the increasingly greater analogy he again and again discovered the limits of cohesion and noncohesion in the generalizations of the primary expression of the theorem. For the series of limits of cohesion and noncohesion from his curve of forces he found the following analogous creations in the equilibrium state of the third point of the system: (1) vertices of an ellipse; (2) vertices in the system of confocal ellipses; (3) whole perimeters in the system of confocal ellipses; (4) whole surfaces in the system of confocal spheroids. The analogy is thereby complete, both from the point of view of the dual character of limits and from the point of view of their vicissitude. In this way the limits of cohesion and noncohesion became that analogous structure that essentially determined Boscovich's direction of research and crowned his endeavor with the spatial conception on the system of confocal spheroidal shells as equilibrium states of the third point of observed system. This conception, together with the established mathematical methodology, deserves to be called Boscovich's "model of atom" from 1748.

Notes

(1) V. Žardecki, "Jedan stav Rudera Boščovića i osnove teorije kvanta", Glas Srpske kraljevske akademije, 185 (1941), pp. 67-82; H.V. Gill, Roger Boscovich, S. J. (1711-1787). Forerunner of modern physical theories (Dublin, 1941), pp. 22-26; Ž. Marković, Rude Bošković, vol. I, (Zagreb, 1968), pp. 436-437; Ž. Dadić, Ruder Boscovich (Zagreb, 1987), pp. 88-89.

(2) R. Boscovich, Dissertationis de lumine pars secunda (= L2) (Rome, 1748), nn. 20-24, pp. 9-12.

(3) L2, nn. 25-32, pp. 12-15.

(4) L2, n. 28, p. 13: "Punctum positum in utrolibet vertice axis utriuslibet erit in limite attractionis, et repulsionis. Sed positum u-

bicunque in perimetro ejus Ellipseos habebit vim, quae ispum urgeat secundum directionem ipsius, versus verticem proximum axis conjugati".

(5) L2, n. 31, p. 14: "Et haec, quae de perimetris dicta sunt, locum habent in superficiebus spheroidum genitarum revolutione facta circa axes transversos".

(6) F. Hund, Geschichte der Quantentheorie (Darmstadt, 1984), pp.59-62, 66.

(7) L2, n. 31, p. 14.

(8) L2, n. 32, pp. 14-15.

(9) R. Boscovich, Theoria philosophiae naturalis (Venice, 1763), n. 236, p. 109.

(10) R. Boscovich, Theoria philosophiae naturalis (Venice, 1763), nn. 223-227, pp. 103-105.

(11) R. Boscovich, De maris aestu (Rome, 1747), n. 90, p. 45.

(12) R. Boscovich, De lege virium in natura existentium (Rome, 1755), nn. 75-110, pp. 29-37.

Bicentennial commemoration of R.G. Boscovich

Milano, September 15-18, 1987

PROCEEDINGS

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