International studies in the philosophy of science 4 (1990), pp. 247-262.

Theories and inter-theory relations in Bošković

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[247] Abstract *During 1745-1755 Bošković explicitly used the concept of scientific theory in three cases: the theory of forces existing in nature, the theory of transformations of geometric loci, and the theory of infinitesimals. The theory first mentioned became the famous theory of natural philosophy in 1758, the second was published in the third volume of his mathematical textbook* Elementorum Universae Matheseos *(1754), and the third theory was never completed, though Bošković repeatedly announced it from 1741 on. The treatment of continuity and infinity in natural philosophy, geometry and infinitesimal analysis brought about inter-theory relations in Bošković's work during his Roman period. The two constructed theories of Bošković, the theory of forces and the theory of geometric transformations, directly influenced the idea for the construction of his third theory. These written theories refer to understanding and effective application of continuity and infinity in natural philosophy and geometry, and this task, according to Bošković, requires methodological support from the theory of infinitesimals.*

Introduction

During the fruitful period (1745-1755) of his professorship at the Collegium Romanum, Rugjer Bošković (Rogerius Josephus Boscovich, Dubrovnik, 1711—Milan, 1787) explicitly used the concept of scientific theory in three cases: the theory of forces existing in nature *(theoria virium in natura existentium),* the theory of transformations of geometric loci *(theoria transformationum locorum geometricorum),* and the theory of infinitesimals or indefinite quantities *(theoria* *infinitesimorum = theoria indefinitorum sive indefinite parva sint, sive indefinite magna).* The theory first mentioned became the famous theory of natural philosophy in 1758, the second was published in the third volume of his mathematical textbook *Elementorum Universae Matheseos* (1754), and the third theory was never completed, though Bošković repeatedly announced it from 1741 on. Here I will explore the way Bošković was comprehending the concept of a scientific theory and inter-theory relations during his Roman period.

The famous theory of forces

Bošković was constructing and enriching his theory of forces from his first idea expressed in the treatise *De Viribus Vivis* (1745) to the final synthesis in his masterpiece *Theoria Philosophiae Naturalis* (1758).[1] He also expressed his second reflections in his letter to Giovan Stefano Conti of 26 February 1762 (Boscovich, 1980, pp. 46-85), and in the notes to the tenth book of Benedikt Stay's poem *Recentioris philosophiae* on Newton's and Bošković's natural philosophy, published posthumously in Stay (1792, pp. 397-513). **[248]**

In *De Viribus Vivis* Bošković evaluated his main conception about forces in nature unequally: as *sententia, theoria,* and *hypothesis.* Before he explained the importance of the principle of continuity and started to trace his curve of forces, Bošković expressed his idea as

> our sentence that leads to a greater simplicity, to the analogy regarding to potentions [= causes of motion] themselves, and to their way of acting. (Boscovich, 1745a, n. 40, p. 31)[2]

There is no doubt what Bošković means by this sentence, because in the same treatise he speaks about sentences of Leibnizians, Anti-Leibnizians, Cartesians, and Newtonians. This means that his *sententia* appears as just another sentence among already deep-rooted sentences, and at the beginning of his discourse it does not explicitly guarantee its value and truthfulness: 'if something more serious is brought up against it, we are prepared to relinquish this sentence and follow the common one' (Boscovich, 1745a, n. 40, p. 31).[3] After having established the basic shape of the curve of forces with the help of a deductive reasoning, as explored by Martinović (1987a), Bošković continues to research the flow of the curve by means of phenomena. In the same research he says: 'And according to the same theory there will be a difference between soft and elastic bodies.' (Boscovich, 1745a, n. 55, p. 41).[4] Here, for the first time, he calls the fundamental deduction of his natural philosophy a theory. In the next step, however, he gives it the value of a hypothesis. Namely, while discussing the problem of the composition and resolution of particles, he begins his explanation as follows: 'since all phenomena in this hypothesis are dependent on the actions of forces in the same way' (Boscovich, 1745a, n. 61, p. 45).[5] Therefore, in the treatise *De Viribus Vivis,* Bošković does not take the definite attitude toward whether his idea about forces is a hypothesis, a theory or just a sentence.

On the contrary, in his next treatise which belongs to the development of the theory of forces, in *Dissertationis de Lumine pars Secunda* (1748), this doubt exists no more. From the very beginning of the treatise Bošković uses the term *theoria* with no exceptions, mentioning in the second paragraph of the treatise *nostra quaedam theoria virium in Natura existentium* (Boscovich, 1748, n. 2, p. 1), and several times later *theoria nostra universa* and *nostra theoria.*[6] It is, of course, not only the term that is being referred to, but also testifies the self-consciousness of the scientist. Bošković notes down the development of the theory:

> Three years ago we only outlined it in the treatise *De viribus vivis, ...;* and soon we will explain it more diffuse together with the principal foundations of mechanics which are either necessary to confirm it or which are derived from it, ... (Boscovich, 1748, n. 2, p. 1).[7]

This intention can refer to the following two works only: Bošković's work synthesis *De Continuitatis Lege* (1754) and his final synthesis *Theoria Philosophiae Naturalis* (1758).

But already in the treatise *De Lumine,* the mutual relationship between the theory of forces and mechanics is stressed, i.e. the theory of forces is evaluated as a successful tool for a reinterpretation of mechanical phenomena researched till then. Bošković explains in which way the theory is proved: the theory is derived positively and directly from the simplest and most widely accepted principles (Boscovich, 1748, n. 3, p. 2);[8] there are direct and sufficiently valid proofs for the theory that are based on the principle of continuity which is widely accepted and confirmed by the ample induction (Boscovich, 1748, nn. 40-41, p. 18).[9] Bošković's judgements that his theory of forces was completely simple and that it had not been stated as an arbitrary hypothesis correspond directly to or are inspired by starting points of Newton's philosophy of science: *Regula philosophandi prima* **[249]** and *Scholium generale* (Boscovich, 1748, n. 40, p. 18).[10] Bošković's research motive (formulated in Boscovich, 1748, n. 40, p. 18) is the reflection itself upon nature *(ipsa Naturae consideratio),* and by no means innovation ardour *(innovandi ardor).* This motive was deepened in the

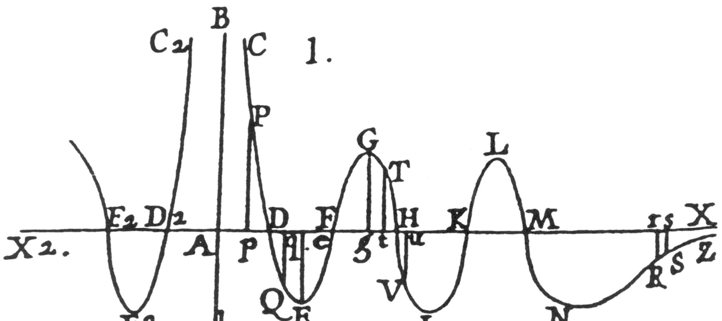

FIG. 1. *Bošković's law of forces, from Boscovich (1748, Fig. 1). Courtesy of the Historical Archives* (Historijski arhiv), *Dubrovnik.*

beginning of the treatise *De Continuitatis Lege.* By Bošković's admission (Boscovich, 1754e, n. 2, p. 3), the construction of the theory was not incited by affection toward fruits *(fructuum amor),* but by gift of nature *(naturae indoles),* and power of argumentation *(vis argumentationis).* How should Bošković's basic motive for researching nature be understood? This is obviously a critical attitude of a scientist who did not burden himself with or become a prisoner of an immoderate wish for the new and for results in scientific work. Such freedom of spirit, present in *De Lumine,* is certainly connected with original spiritual heritage of the religious community to which Bošković belonged.[11] Besides, its appearance in the treatise *De Continuitatis Lege* was the announcement of the first troubles Bošković had because of his theory in the community of Roman Jesuits and the form of an anticipated defence before the 'scandal Benvenuti' in the Collegium Romanum broke out vehemently (Villoslada, 1954, pp. 242-243; Marković, 1968, pp. 290-292; Martinović, 1986a, pp. 10-11; Dadić, 1987, pp. 63-64).

Finally, in the treatise *De Lumine,* Bošković formed a systematical explanation of his theory for the first time:

(1) points of matter endowed with certain forces *(materiae puncta praedita viribus quibusdam)* (Boscovich, 1748, n. 4, p. 2);

(2) law of forces expressed by a continuous curve *(virium lex per curvam regularem)* (Boscovich, 1748, n. 5, p. 2), whose graph is shown in Fig. 1;

(3) application to the constant and permanent order of natural phenomena and to the structure of matter *(constans et permanens ordo phaenomenorum, et textura corporum)* (Boscovich, 1748, n. 9, p. 4).

Bošković explained the theory in the same way in all its explications between 1745 and 1758. Therefore this explanation has been the basic form of reception of Bošković's theory of forces in scientific circles during the past two centuries. However, we must distinguish between a systematic exposition of a constructed theory and a genesis of the same theory. This can especially be applied to

Bošković's theory of forces, as I proved by doing research into Bošković's thought horizon during 1745-1748 (Martinović, 1987a, pp. 93-99).

In 1748 Bošković also wrote the treatise *De Materiae Divisibilitate et Principiis Corporum.* He published it in 1757, when he added an introduction and notes to it. The authentical text from 1748 and the added texts from 1757 differ considerably regarding the evaluation of Bošković's results in the research of nature. The text of the treatise contains **[250]** an explanation of how Bošković was inspired with ideas from *Query 31* of Newton's *Opticks; it* is the most extensive explanation of this kind in the complete works of Bošković.[12] At the same time, the text gives evidence of Bošković's years of persevering considerations before he reached the original solution: 'Moreover, I made progress in that I reduced primary properties of matter to an unique principle and proved this principle by right ratiocination.' (Boscovich, 1757, n. 3, p. 138).[13] Bošković named this achievement the sentence. Later on in the text (Boscovich, 1757, n. 12, p. 151) *punctorum indivisibilium sententia* was discussed. While discussing Newton's ideas from *Query 31,* he seems to become more careful because he uses the terms *Nevvtoniana sententia* and *mea sententia* (Boscovich, 1757, n. 82, p. 237). Use of the term *sententia* can be explained in two ways. Either Bošković finished a draft of the treatise *De Materiae Divisibilitate* before the final editing of the treatise *Dissertationis de Lumine pars Secunda,* which is still an open problem according to Martinović (1986a, pp. 8-9), or, on the contrary, the readoption of the term *sententia* is a retreat from the objections made after solemn defence of Bošković's treatise on light held at the Collegium Romanum. On the other hand, Bošković consequently and resolutely uses the term *theoria* in the introduction and notes written nine years after the text itself. In fact, according to Bošković's opinion, expressed in the introduction, the treatise gave him an opportunity to explain and expand his theory of general physics (Boscovich, 1757, p. 131).[14] Bošković reminds us that although there were many objections to his sentence printed during the period between 1745 and 1757,[15] he could not find or notice any among them which would be a serious one or one that cannot be disputed. 'Therefore,' Bošković concluded, 'this [sentence of indivisible points] is not an arbitrary hypothesis, but a theory deduced from truthful principles and proved.' (Boscovich, 1757, p. 151).[16]

In treatises from 1748 Bošković applied his theory of forces in order to explain numerous physical phenomena. He promoted this approach later as well. In the treatise *De Centro Gravitatis* (1751) he pointed out that his theory of forces depending on distances *(theoria virium a distantiis pendentium)* quite simply explains density of light (Boscovich, 1751, n. 107, p. 27). In the treatise *De Continuitatis Lege* he announced that the treatise *Synopsis Physicae Generalis* by Carlo Benvenuti, a professor of metaphysics at the Collegium Romanum in the course of the academic year 1753-1754 in accordance with Iparraguirre (1954, p. 329), would be published soon. The fourth part of this treatise (Benvenutus, 1754, pp. 38-81) was an attempt to reinterpret the general physics by means of Bošković's theory of forces. Bošković actively took part in the genesis of Benvenuti's work. Moreover, he included the paragraphs of Benvenutus (1754, nn. 146-152, Fig. 12, pp. 56-59) into his *Theoria* as the fifth supplement (Boscovich, 1763, nn. 86-92, Fig. 75, pp. 293-296). The results Bošković obtained in August 1754, as well as his fruitful relations with Benvenuti, encouraged him to consider for the first time general physics *(universa Physica)* as a field of the application of his theory (Boscovich, 1754e, n. 158, p. 73).[17] Till then he had restricted himself mostly to the mechanical properties of matter.

However, Bošković did not only expand the field which his theory of forces can be applied to, but also questioned its foundations once more. In *De Continuitatis Lege* he comprehensively proves the principle of continuity which is the principal foundation ofhis universal analysis *(praecipuum universae analyseos nostrae fundamentum)* (as stated by Boscovich, 1754e, n. 3, p. 4;

Boscovich, 1755b, n. 1, p. 3), in order to explain succinctly at the conclusion of his treatise (Boscovich, 1754e, nn. 158-174, pp. 73-80) the deduction of his theory from the law of continuity.[18] As in the treatise *De Viribus Vivis,* here, too, deduction starts by considering the collision between two bodies as the decisive physical situation which can successfully be explained by the action of infinite force on infinitely small distance. In order to exclude jump change it is not enough to presume, like **[251]** Leibnizians did, that there are only soft and elastic bodies and that by their collision the continuous deformation of body or surface in contact arises. Hence it follows that there are indivisible and non-extended points of matter, mutually separated by a certain interval. Parallel to this argumentation, Bošković shaped the curve of forces, which shows forces depending upon distances between particles, and described the main characteristics of its flow. According to Bošković's opinion, this curve represents, in spite of its complicated trace (Fig. 1), the simplest law *(simplicissima lex),* as suggested in Boscovich (1754e, n. 169, pp. 78-79). This claim corresponds with *analogia et simplicitas naturae* as the fundamental epistemic starting point of Bošković, but only this claim reveals why Bošković opened the controversy on the simplicity of the straight line in 1747, as explained in Martinović (1986b, pp. 176-179). Bošković also believes that his unique law *(unica lex)* has an amazing capability to explain all general and particular properties of bodies. The application of the theory to physics completes Bošković's chain of deductive reasoning.

Bošković regularly and with pride names this deduction from the law of continuity *nostra theoria* and *nostra virium theoria* (Boscovich, 1754e, nn. 158-159, p. 73; n. 160, p. 74; n. 169, p. 79; n. 173, p. 80). However, the term *nostra sententia* also occurs in the treatise, but it is reserved only for the study of space and time.[19] Thereby Bošković obviously states that his analysis of forces in nature and his analysis of space and time do not show equal certainty. Spacial and temporal relations should be studied further and established more precisely, and until then (Boscovich, 1755a) they should be considered separated from the whole of Bošković's theory of forces.

In the treatise *De Lege Virium in Natura Existentium* (1755) Bošković synthesized his research into the continuous curve of forces which has been accepted among contemporaries as *curva Boscovichiana* (Fig. 1). On this occasion, too, he calls his interpretation of forces in nature 'our theory of forces existing in nature' *(nostra theoria virium in natura existentium),* as quoted in Boscovich (1755b, n. 1, p. 3). This syntagm faithfully described Bošković's attitude from 1748 till the final revision of the theory during 1757-1758. That revision marked a turning point in Bošković's understanding of the theory of forces which manifested itself in a special way in introductions to Bošković's works from this period. Even in the first sentence of the introduction to the later edition of the treatise *De Materiae Divisibilitate et Principiis Corporum* (Boscovich, 1757, p. 131) Bošković calls the theory of forces 'my theory of general physics' *(mae theoria Physicae Generalis).* Lastly, while finishing the final editing of his work *Theoria Philosophiae Naturalis* in February 1758, Bošković characterized it in the epistle dedicated to Christopher de Migazzi (Boscovich, 1763, pp. IX—X) as 'the general contemplation upon Nature' *(universa Naturae contemplatio),* 'theory of general physics, in fact a completely new theory' *(Universae Physicae Theoria, et nova potissimum Theoria),* and 'a new kind of general natural philosophy' *(novum quoddam Universae Naturalis Philosophiae genus).* This is to say that his interpretation of forces became the interpretation of the whole nature. Force, understood in Bošković's way, became the only key for the explanation of all phenomena in nature. Finally, this conception resulted in the syntagm *theoria philosophiae naturalis* in the very title of Bošković's masterpiece. While constructing his theory of forces during 1745-1758, Bošković matured in the evaluation of his theory from a sentence in the 1740s to the theory of natural philosophy.

The theory of geometric transformations

From 1747 on Bošković considered the idea of constructing a theory of geometric transformations based on his study of conic sections. This proves his intention announced in the treatise *De Maris Aestu:* **[252]**

> Namely, this topic requires a complete and even more extensive treatise which we, among other treatises, leave for our *Elements of conic sections,* which has in greatest part been prepared and anticipated only the final brush-up. In this treatise we will reveal amazing characteristic, marvelous transformation, and nexus of geometric loci, as well as the arcana of infinity which are surely necessary, if we admit infinity, and which considerably exceed all human power of comprehension. (Boscovich, 1747, n. 90, p. 45)[20]

Indeed, Bošković constructed the theory in the last quarter of 1753 and included it into the third volume of his mathematical textbook *Elementorum Universae Matheseos* together with *Sectionum conicarum elementa.* He did not finish the third volume until January 1754, as is witnessed by the letter he wrote to his brother Božo in Dubrovnik on 22 January 1754, in which he announces its publication in the following week, convinced that this is his first work done with full care.[21] In this letter Bošković explicitly explained why it took him so long to finish the third volume:

> At the end of my Conic Sections it seemed appropriate to add a treatise on the transformations of geometric loci and on the infinite, which has grown so much under my pen, that it exceeds a third of the volume. (Truhelka, T-25, VIII, 43)[22]

This was the treatise *De Transformatione Locorum Geometricorum*

(Boscovich, 1754b). While writing the preface to the third volume of *Elementorum Universae Matheseos* (Boscovich, 1754c, p. XXI) Bošković for the first and only time called his exposition about the transformations of geometric loci, *transformationum theoria.* Here, too, as with the theory of forces, Bošković's consciousness about the theory follows from his second evaluation of the realised geometric results. In spite of this, Boscovich (1754b) has not been systematically studied to date as a constructed theory of Rugjer Bošković. Researchers into Bošković's contributions to mathematics either have reviewed Boscovich (1754b) concisely from the point of view of Bošković's introduction to the third volume of *Elementorum Universae Matheseos* (Marković, 1968, pp. 284-286), or they have dealt with some special questions (Majcen, 1921; Kolman, 1962, pp. 92-95; Stipanić, 1975, pp. 121-134; Martinović, 1986b, pp. 172-174; Martinović, 1987b). Bošković's geometric theory has remained in the shadow of his famous theory of forces.

The treatise *De Transformatione Locorum Geometricorum* (Boscovich, 1754b) can be thematically divided into two parts. In the first part, including Boscovich (1754b, nn. 673-758, pp. 297-367), 'a certain material prepared for a new building' *(materia quadam novi cujusdam aedificii praeparata)* is exposed, as pointed out in Boscovich (1754c, p. XXI). The second part, including Boscovich (1754b, nn. 759-886, pp. 367-468), contains the very theory of geometric transformations in accord with Bošković's views how a geometric theory should be constructed: first, the definition of the two-fold analogy, and then 11 canons. The definition and the canons form the axiomatic system of Bošković's geometric theory. The material gathered for the theory and the very theory are equally structured. It can be followed from the introductory paragraphs of both parts of the treatise: from n. 673 where the presentation of the material begins, and from n. 759 that represents the programmatic introduction to the general theory of the transformations of continuous curves.

Bošković's approach to the research into the transformations of geometric loci is constant throughout the treatise. While choosing the material and, also, while constructing his theory of geometric transformations, Bošković attempted to comprehensibly conceive 'the law of geometric continuity' *(continuitatis geometricae lex)* and tried to explain 'some mysteries of infinity' *(quaedam infiniti mysteria)* seeing in them 'a wonderful ability of **[253]** geometry' *(mira Geometriae indoles)* .[23] The same intention is obvious in some other places. The subtitle of the treatise is *ubi de continuitatis lege, ac de quibusdam Infiniti mysteriis.* In the introduction paragraph of the treatise (Boscovich, 1754b, n. 673, p. 297), as well as in the preface to the third volume of Bošković's *Elementorum Universae Matheseos* (Boscovich, 1754c, pp. XVII—XX), geometric continuity and geometric infinity are considered in their mutual relationship: the appearance of the mysteries of infinity actualizes the demand that the continuity should be everywhere preserved and strictly persevered, and vice versa, where the behaviour of a curve is described with the help of continuity, the mysteries of infinity still appear. Such a relationship between continuity and infinity is expected wherever the wonderful gift of geometry is mentioned, and this property is said to be obvious in every transformation of geometric loci.

Bošković's clearly defined approach to the study of geometric transformations can be followed through the basic thematic division of the treatise. The first part contains a survey of all qualitative forms of behaviour of continuous curves in geometric transformations, including the behaviour of continuous curves in infinity and in zero. Bošković simultaneously investigates the increase of a geometric object in infinity as a way of existence of the infinite and its vanishing as a way of existence of the infinitely small. Bošković's approach has been known since his early mathematical treatise *De Natura et Usu Infinitorum et Infinite Parvorum* (Boscovich, 1741, nn. 12-16, pp. 7-9), and here, it is a remarkable beginning from his study of a straight line as a continuous geometric creation (Boscovich, 1754b, n. 695, pp. 314-315). The second part of the treatise *De Transformatione Locorum Geometricorum* is already the realization of a new programme of research into continuity and infinity in geometry.

The core of Bošković's research programme makes the axiomatic system which consists of the definition of the analogous geometric creation and of 11 canons for the transformations of geometric loci. Bošković is aware of the originality of his 'new building' because he speaks about his treatise:

> It contains, however, a lot of things which it seems very worthy to know, but what I have not met with it in another place, and a lot of things which can be often occurred elsewhere, but what I have not found anywhere reduced to sure canons and studied by the geometric method. (Boscovich, 1754c, p. XVIII)[24]

Still, he mentions that a similar system could be found in some unknown, very old papers (Boscovich, 1754c, p. XVIII).[25] This confirms once again that he founded his system of canons with the help of the synthetic geometric method.

The construction of the axiomatic system of Bošković's theory commences with the definition of the primary and secondary analogy:

> 760. First of all, the points determined in the same way in both states of the same geometric construction, i.e. in the state before and after the transformation, are called the *analogous* points. They are, of course, determined by the intersection of the same geometric loci: straight lines with other straight lines, circle, perimeter of the conic section, lines defined by such intersection according to the same law.... Analogous, however, we call lines terminated by two analogous points, surfaces terminated by analogous lines, solids terminated by analogous surfaces....
>
> 761. Then, we distinguish two kinds of this analogy. One is *primary*

> and complete, when after the transformation, the direction of defined quantity is left, or it is changed with an even number of changes. The second kind of analogy is **[254]** called *secondary* when the direction of quantity is changed suddenly or with an odd number of changes, thus it can be called the *antianalogy*.... (Boscovich, 1754b, nn. 760-761, pp. 368-369, emphases in original).

In Bošković's supplemental explanation within the preface to the third volume of his *Elementorum Universae Matheseos* (Boscovich, 1754c, p. XXI), this definition is called the definition of the duplex analogy *(duplicis analogiae definitio).*

This fundamental definition is followed by 11 canons. What are Bošković's canons characterized by? It is clearly remarkable in their statements how Bošković differentiates in every solved geometric problem the following elements: proposition *(enuntiatio),* proof *(demonstratio),* and solution *(solutio).* A canon indicates what happens in a transformation with these three forms of the geometric problem, i.e. what is changed in proposition, proof and solution, if it is changed at all. A striking example is offered by the first canon:

> 764. Canon I. If the quantities, which express the solution of a problem or the proposition of a theorem, remain all analogous in the sense of the first [primary] analogy after the transformation, and there is no transition through infinity, then the solution, proposition and proof remain the same, changed neither really nor literally. But, if we conceive that some of them are transmited through infinity and are coupled and connected in this same infinity, it leads to the both sides at last: in those quantities that depend only on direction everything remains in the same manner; but in those pertinent to the magnitude it is obliged to estimate that ratio of theirs which originates from the law by which they are determined, and [which

ratio] is wholly analogous to that one they would get if they not transmit through infinity. (Boscovich, 1754b, n. 764, p. 373)

The canons hold in the universal geometry (Boscovich, 1754b, n. 759, p. 368).[26] The question arises how Bošković proves the canons when he points out their general validity? Bošković claims in the preface (Boscovich, 1754c, p. XXIII) that 'Some canons are proved exactly.',[27] and then he describes the character of these proofs by terms such as *exempla, applicatio,* and *usus.* Are examples, application, and use the same as exact proof? Obviously, when constructing the theory of geometric transformations, Bošković acts differently from when he examined the fundamentals of infinitesimal calculus at the beginning of his mathematical career, as explored in Martinović (1988). In *De Natura et Usu Infinitorum et Infinite Parvorum* (1741) he questioned the nature of basic calculus concepts and studied counter-examples, such as absurdity of the actual infinite for geometric extension, and inflexion point of cubic parabola, to such a degree that it became an epistemic barrier for the use of the infinitesimal in his mathematical investigations. This was why he did not systematically research into any field of the application of infinitesimals. On the contrary, the nature of basic geometric quantities in Bošković's theory of geometric transformations is not questionable thanks to their Euclidean origin, and it is not questionable from the beginning of constructing the theory. Therefore, there is a great possibility of their use in geometric transformations. The basic geometric quantities become questionable only in transformations, as a rule in connection with the infinite. A typical example, taken from Bošković's tenth canon, is a straight line understood as an infinite circle, and this idea has a long Neoplatonic tradition.

According to Bošković's conception, any transformation of geometric locus can successfully be described by means of the transformations of basic geometric objects: quantities, proportions, and angles. Thus it is possible to classify the first nine canons of Bošković's theory of the transformations into canons of quantities, canons of proportions, **[255]** and canons of angles. The last two canons describe specific situations. The tenth canon considers the straight line as an infinite circle, and the eleventh discusses the comparison between the geometric infinites. Therewith two kinds of changes, which occur in geometric constructions as a result of transformations, are comprized in the theory: (1) appearance of impossible or imaginary quantities; (2) vanishing of the point in infinity, either as an intersection of straight lines, or as a centre of a circle, or, quite generally, as a solution of a geometric problem. Bošković thus attempts to construct such a system of canons which would completely describe all the changes arisen from geometric transformations. However, it has to be researched into how far Bošković succeeded in constructing his system of canons.

The theory of infinitesimals

Bošković announced the theory of infinitely small quantities or the theory of indefinite quantities several times, starting from his last sentence in the inaugural mathematical treatise (Boscovich, 1741, p. 12) at the Collegium Romanum up to the preface of his mathematical textbook (Boscovich, 1754d) and to the scientific report on determination of the shape of the Earth (Boscovich, 1755c, n. 259, p. 483). However, within his evaluation of Euler's work, *Introductio in Analysin Infinitorum,* in the letter to his student Francesco Puccinelli, dated 15 November 1763, he asserted that he was not able to become master of this mathematical discipline any more.[28] It was then that he gave up the year-long intention to expose the infinitesimal calculus in the fourth volume of *Elementorum Universae Matheseos.* It is certain that in 1763 he did not feel compelled to round off his mathematical textbook with the last planned volume, *Elementa Infinitorum, et Infinitesimorum,* because from 1760 he was not a professor of mathematics at the Collegium Romanum. The manner in which he presents the material in the third volume allows us to presume that he would choose an untrodden path for the exposition of the infinitesimal calculus for his students. However, from the numerous announcements of the fourth volume during the period between 1741 and 1755 many details can be singled out. It is possible to judge the complexity of its announced content and find out the basic conception in constructing the theory of infinitesimals. This is to say that it is possible to write the prehistory of an unwritten theory.

Infinitesimorum theoria was included in Bošković's research plans during the period of intensive construction of his theory of forces in 1748. Bošković wanted to show that 'also the complete theory of infinitesimals depends only on the exclusion of leap,' i.e. that the theory of infinitesimal quantities can be founded by means of the principle of continuity.[29] The motive which inspired Bošković's announcement was the very congruence between the structure of the theory of forces and the theory of infinitesimals. As the principle of continuity is the formative principle of Bošković's theory of forces, in the same way the continuity of curve should have been the foundation stone of the conceived theory of infinitesimals.

In 1753 the theory of infinitesimals gained additional momentum from Bošković's investigations of geometric transformations. In the programmatic introduction of his theory of transformations of geometric loci Bošković pointed out:

> Indeed, certain mysteries of infinity will sundry times occur and they grow to such an extent that they finally point to the impossibility of the extension infinity and lead us toward the theory of indefinite quantities, either indefinitely small or indefinitely large quantities, which will be examined in another work. (Boscovich, 1754b, n. 759, p. 368)[30] **[256]**

In addition, the theory of transformations of geometric loci was conceived as integral research into continuity and infinity in geometry. The role which was assigned to the infinite incited Bošković toward a new theory, which this time he named *indefinitorum theoria.* Bošković, thus, used his favourite expression, which he took over from Leibniz and which unites concepts of the infinite and the infinitely small as manifestations of the potential infinite. In the introduction to the third volume of *Elementorum Universae Matheseos,* Bošković gave an outline of his theory of indefinite quantities in the planned fourth volume:

> Another [work] will follow that will deal with infinite or infinitely small [quantities] which are indefinite to me. I will explain their nature, distribute their orders, lecture on elements proving them by geometric rigour, and then I will turn toward general properties of curves, cusps, inflexions, infinite branches, contacts, osculations, evolutes, theory of the maxima and minima. I will also explain the other things, and deduce and prove singular properties of the special and most useful curves. (Boscovich, 1754c, pp. XXV—XXVI)[31]

In his later treatises, too, Bošković referred to topics which he will deal with in his theory of infinitesimals. These topics usually refer to continuity and infinity of geometric curves. While discussing the nature of infinite branches of curves and their connection in infinity, for example, the nature of the parabola, hyperbola, logistica, logarithmic spiral, and Nicomedean conchoid, in the treatise *De Continuitatis Lege,* Bošković was aware that such research belongs to a special mathematical discipline which he named geometry of the infinitesimal and infinite quantities *(infinitesimorum et infinitorum Geometria)* (Boscovich, 1754e, n. 96, p. 43). Therefore, differential geometry should have been the subject of the fourth volume of Bošković's *Elementorum Universae Matheseos.* Bošković's interest in the theory of infinitesimals was also supported by his investigations in other fields of science, one of which, that I would especially like to present here, came from geodesy. While determining the shape of the Earth in the scientific report *De Litteraria Expeditione per Pontificiam Ditionem* (1755), Bošković often compared the geometric and infinitesimal method and preferred to apply the geometric method. However, he was not exclusive regarding the choice of the mathematical method. While researching into the shape of the Earth from the equilibrium of a fluid rotating on its own axis, he pointed out the priority of the elementary infinitesimal calculus over geometric proofs and used the infinitesimal formula for the subnormal (Boscovich, 1755c, nn. 61-63, pp. 408— 409).[32] While exploring the arc of a meridian in cases of unequal density of the Earth, he discussed the concept of the osculating circle, and particularly anomalous points at which the circle of osculation does not exist (Boscovich, 1755c, nn. 258-266, pp. 482—486).[33] Finally, Bošković worked on the general solution of the problem of determining the curve of the meridian for a given series of degrees. This time he decided to construct the curve by means of geometry, although in the analysis of the problem he mentioned osculating circles and evolutes (Boscovich, 1755c, nn. 306-309, pp. 502-504).[34] Bošković studied irregularity of the curve of equilibrium *(irregularitas curvae aequilibrii)* with special attention, examining what happens with osculating circle at this point (Boscovich, 1755c, n. 322, pp. 509-510).

All given examples are focused on the framework of Bošković's theory of infinitesimals. This theory would obviously have been based on the concepts of continuity and infinity as they are defined for geometric curves. It would expose the complete material which is required by applications of the infinitesimal calculus in geometry. The applications would surely have contained chapters about the tangent and normal curve, determination of **[257]** maxima and minima, and the singular points of curve, as well as about the osculating circle and evolute. The theory would have ended with research into some important curves. Among these curves Bošković would very probably have included cycloid and logistica, for which he wrote a comprehensive geometric review in

Boscovich (1745b). Although this framework of his theory of infinitesimals was never realized, Bošković solved several mathematical problems in terms of infinitesimal analysis. I would like to stress the significance of his published results: the problem of the solid of greatest attraction acting at a point of axis of this solid, solved with the help of geometric and infinitesimal methods in Boscovich (1743); the determination of the geometric form of the cells of bees as a problem of finding the minimum surface, given in geometric and analytic terms, published in Boscovich (1760); and four general differential equations of spherical trigonometry, based on the differential changes of a spherical triangle within the project of the verification of astronomical instruments, introduced in papers sent to the Academie des Sciences (Paris) in 1772, and published in Boscovich (1785). The same methodological choice can be found in Bošković's scientific correspondence. As pointed out by Baldini and Nastasi in Boscovich (1988, pp. 18-19) and also by Homann (1989, pp. 562-563), Bošković's *Memorietta,* attached to his letter to Anton Mario Lorgna of 24 May 1768 and recently published in Boscovich (1988, pp. 56-71), sketched how curves of higher than second order can be studied with the help of differential calculus.

Conclusion: the role of continuity and infinity in inter-theory relations

All three theories of Rugjer Bošković, the two constructed and one only conceived, are focused on the same problem, that of continuity and infinity, which has been an important topic of scientific and philosophical investigations from Aristotle's formulation of the problem in *Physics* to the present day. Bošković's theories solved, or at least tried to solve, the same problem in three different fields: natural philosophy, geometry, and infinitesimal analysis.

Bošković's theory of forces is founded on the principle of continuity and its qualitative consequence, which is the continuous curve of forces *(curva Boscovichiana).* It describes, among other things, the action of infinite repulsive force on infinitely small distances (see the arc CD in Fig. 1). Bošković was convinced that with the help of his curve of forces he succeeded in explaining the circle of physical phenomena that formed the Newtonian integral heritage. This realm of physical phenomena was designated by the term ‘philosophia naturalis’ in the title of Newton’s masterpiece *Philosophiae Naturalis Principia Mathematica.* In fact, as explained by Westfall (1983, p. 459), Newton insisted on the word ‘mathematical’ in the title of his work:

> In the preceeding books I have laid down the principles of philosophy; principles not philosophical but mathematical: such, namely, as we may build our reasonings upon in philosophical inquires. (1934, p. 397)

The question arises whether Bošković's principle of continuity was a mathematical principle in Newtonian sense. Indeed, Bošković explicitly considered Newton’s ideal in the investigation of nature, expressed in *Opticks* in the following way:

> On the other hand, to deduce from the phenomena of Nature two or three general principles of motion and to explain how the properties and actions of all corporate things follow from these principles, this would indeed be a mighty advance in philosophy, even if the causes of those principles had not at the time been discovered. (Newton, 1964, p. 261) **[258]**

Newton had selected three principles: gravitation, the cause of fermentation, and the cause of the cohesion of parts, in order to explain the motion of particles and other known natural phenomena. In Boscovich (1748, n. 58, pp. 24-25), Newton’s *maximè desiderandum* from *Opticks* turned into Bošković’s law of continuity *(lex continuitatis)* as the unique principle in the research of nature.[35] Later, in the epistle to Christopher de Migazzi (Boscovich, 1763, p. VII), Bošković characterized his *Theoria Philosophiae Naturalis* as ‘a little book ...

containing principles of natural philosophy' *(libellum ... Naturalis Philosophiae principia continentem).* Obviously, Bošković evaluated the principle of continuity as a mathematical principle in the Newtonian sense of the word. The same principle, formulated in Boscovich (1745a, n. 45, p. 35) as the passage from one magnitude to another through all intermediate magnitudes of the same class, should be the foundational principle of Bošković's theory of infinitesimals, as proposed by Boscovich (1748, n. 54, p. 23). However, the principle of continuity never reached the mathematical rigour Bošković expected and requested for mathematical theory. A qualitative approach was sufficient for the successful construction of the theory of forces in the 18th century, but not for the construction of the theory in the field of infinitesimal analysis. And inversely, Bošković's theory of forces could not reach a quantitative stage without a methodological support of constructed theory of infinitesimals.

On the other hand, Bošković's theory of transformations of geometric loci is the realization of a new programme of research into the continuity and infinity in geometry. Bošković constructed the axiomatic structure which completely described all qualitative forms of behaviour of continuous curves in geometric transformations, including the behaviour of curves in the infinitely distant point. Here the basic geometric quantities, such as segments, proportions and angles, are not questionable thanks to their Euclidean origin, but they become questionable only in transformations, as a rule in connection with the infinite. A typical example is the nature of infinite branches of curves and their connectedness in the infinite. Already the attempt to establish the continuity of curves in the infinite has provoked the theory of indefinite quantities. Bošković never proposed the axiomatic structure in the field of infinitesimal calculus, as he established it in the case of geometric transformations. Therefore, his theory of geometric transformations included only qualitative description based on the synthetic geometric method.

The treatment of continuity and infinity in natural philosophy, geometry and infinitesimal analysis led to inter-theory relations in Bošković's work during his Roman period. The two constructed theories of Bošković, the theory of forces and the theory of geometric transformations, directly influenced the idea for his third theory. These written theories refer to understanding and effective application of continuity and infinity in natural philosophy and geometry, and this task, according to Bošković, requires methodological support from the theory of infinitesimals.

Notes

[1] Cf. Boscovich (1763, n. 5, p. 3): 'What has already been published relating to this Theory is contained in my dissertations, *De viribus vivis,* issued in 1745, *De Lumine,* 1748, *De Lege Continuitatis,* 1754, *De Lege virium in natura existentium,* 1755, *De divisibilitate materiae, & principiis corporum,* 1757, and in my *Supplementa* to Benedikt Stay's *Philosophiae versibus traditae,* of which the first volume was published in 1755.' See also Bošković's footnote in (Stay, 1792, p. 397): 'Here now [Stay in tenth book of his poem *Recentioris philosophiae*] leaves Newton and clears the way to my theory, which I had exposed in many dissertations while our [poet, i.e. Stay] wrote this poem; first indeed in the treatise *De Viribus vivis (On living forces),* then in 1748 in the treatise printed after some [eleven] years in the collection of scientific essays in Lucca (Memorie sopra la Fisica, e Istoria naturale di diversi Valentuomini, Vol. 4) under the title *De materiae divisibilitate, &* **[259]** *principiis corporum (On the divisibility of matter, and on the principles of bodies),* and also in the treatise, *De Lumine (On light) ,* printed in the same year 1748. Next, I added the treatise *De Lege Continuitatis (On the law of continuity)* in 1754, and the treatise *De Lege virium in Natura existentium (On the law of forces existing in Nature)* in 1755. Finally, I arranged it all more orderly in the work of appropriate length

entitled *Philosophia Naturalis redacta ad unicam legem virium in Natura existentium (Natural Philosophy reduced to the unique law of forces existing in Nature)* [sic!] which is printed in Vienna in 1758, and again published in Venice in 1763.' (translated by I. M.) cf. the titles of Boscovich (1758) and Boscovich (1763).

[2] See Boscovich (1745a, n. 40, p. 31): 'remanet, ut nostram sententiam quandam aperiamus, quae majorem etiam simplicitatem inducit, et analogiam circa potentias ipsas, et eorum agendi modum.' See also Boscovich (1745a, n. 43, p. 33): 'Sed ut ad fundamentum nostrae hujus sententiae deveniamus.'

[3] 'paratissimi tamen si gravius quidpiam contra ipsam proferatur nobis, eandem deferere, et communem sequi.' (Boscovich, 1745a, n. 40, p. 31.)

[4] 'Et quidem ex eadem theoria, et mollium corporum discrimen habebitur ab elasticis.' (Boscovich, 1745a, n. 55, p. 41.)

[5] 'At quoniam in hac hypotesi phaenomena omnia ex earum virium actionibus pendentia eodem modo se haberent;' (Boscovich, 1745a, n. 61, p. 45).

[6] 'theoria ipsa' (Boscovich, 1748, n. 3, p. 2); 'ex eadem admodum simplici theoria' (Boscovich, 1748, n. 40, p. 18); 'tota positiva et directa probatio hujus theoriae' (Boscovich, 1748, n. 41, p. 18); 'duae potissimae difficultates objici possunt contra hujusmodi theoriam' (Boscovich, 1748, n. 54, p. 22); 'Sed ea omnia, et universam hanc theoriam nostram multo diligentius excolemus, et propugnabimus brevi longiore opere, ...' (Boscovich, 1748, n. 54, p. 23); 'Verum, quae ad hanc difficultatem dissolvendam conducerent jam olim Nevvtonus proposuit in Optica quaestione ultima, ubi simul nobis theoriae hujus nostrae universae occasionem dedit.' (Boscovich, 1748, n. 56, p. 23); 'Hic quidem omnino videre est vestigia quaedam, et prima veluti semina theoriae nostrae' (Boscovich, 1748, n. 57, p. 24); 'quae sane omnia ex nostra theoria prorsus necessario consequuntur.' (Boscovich, 1748, n. 72, p. 30.)

[7] 'Eam tribus ab hinc annis adumbravimus tantummodo in dissertatione de Viribus Vivis, ...; multo autem fusius una cum praecipuis mechanicae fundamentis vel ad eam confirman necessariis, vel ex ea deductis proponemus brevi, ...' (Boscovich, 1748, n. 2, p. 1).

[8] 'tum illa ipsa fundamenta indicabimus, ex quibus eadem, quantum in re Physico-Mathematica licet, positive et directe ex simplicissimis, et jam communissimè admissis principiis deducantur.' (Boscovich, 1748, n. 3, p. 2.)

[9] 'Verum non desunt directae, et satis validae probationes, quae ipsam evincant, ...' (Boscovich, 1748, n. 40, p. 18); 'Tota postiva et directa probatio hujus theoriae innititur huic principio: *In Natura nihil fieri per saltum, ...* Hoc principium passim jam admittitur, et amplissima inductione comprobatur, ...' (Boscovich, 1748, n. 41, p. 18).

[10] 'Plurima alia proferri possent ex eadem admodum simplici theoria deducta, sed haec ipsa tanti ponderis sunt; ut si nihil aliud, nisi hypothesim arbitrariam proferremus;' (Boscovich, 1748, n. 40, p. 18).

[11] Cf. 'Principium, et fundamentum' (Loyola, 1953, p. 42): 'quapropter necesse est facere nos indifferentes erga res creatas omnes, quantum permissum est libertati nostri liberi arbitrii, et non est ei prohibitum, ...'

[12] Cf. Boscovich (1757, n. 3, pp. 137-139; n. 19, pp. 164-167; n. 55, pp. 208-209; nn. 81-89, pp. 235-247) with ample quotations from Newton's *Opticks.*

[13] 'quanto ulterius in primariis ipsis materiae proprietatibus ad unicum principium revocandis, et principio ipso recta ratiocinatione demonstrando progressus sim, ...' (Boscovich, 1757, n. 3, p. 138).

[14] Cf. 'Praefatio' (Boscovich, 1757, p. 131): 'et ea ipsa mihi occasio extitit illustrandae, ac extendendae meae theoriae Physicae Generalis, quam proposueram anno 1745. In 'Dissertatione de Viribus Vivis.'

[15] 'Sunt multa contra ipsam theoriam huc usque objecta typis edita, ...' (Boscovich, 1757, p. 132); 'apud alios, qui meam hanc oppugnarunt sententiam post annos jam 12;' (Boscovich, 1757, n. 138, p. 139, adnotatio (a)).

[16] 'ut adeo haec [punctorum indivisibilium sententia] non hypothesis arbitraria sit, sed theoria e genuinis principiis deducta, ac comprobata.' (Boscovich, 1757, n. 12, p. 151, adnotatio (a).)

[17] 'potissimum quae pertinent ad ipsam theoriam virium illustrandam, et ejus usum per universam Physicam latissime patentem' (Boscovich, 1754e, n. 158, p. 73).

[18] 'Haec nostrae theoriae summa; en ejus deductionem a lege Continuitatis.' (Boscovich, 1754e, n. 160, p. 74.) For a detailed commentary on Bošković's approach, see Stipanić (1975, pp. 154-158).

[19] 'Illud unum hic addemus, quod ad spatii ideam pertinet in hac nostra sententia realem Mathematice continuam extensionem, a corporibus excludente.' (Boscovich, 1754e, n. 171, p. 79); 'in nostra sententia' (Boscovich, 1754e, n. 174, p. 80).

[20] 'Integram hic quidem locus, et quidem fusiorem dissertationem requirit, quam nostris Sectionum Conicarum elementis maximâ jam ex parte diggestis, et ultimam tantummodo manum desiderantibus **[260]** reservamus inter alias multas, qua miram Locorum Geometricorum indolem, miram transformationem, et nexum, et Infiniti arcana omnino necessaria, si Infinitum admittatur, at omnem humanum captum longe excendentia proferemus.' (Boscovich, 1747, n. 90, p. 45.)

[21] Rugjer Bošković to Božo Bošković, 22 January 1754 (Truhelka, T-25, VIII, 43): 'Spero, che uscirà la Settimana, che viene, e posso dire, che è la prima mia opera fatta con tutta l'attenzione, e son persuaso, che avendo molte cose nuove, avrò dello spaccio, e del corso.' See also Marković (1968, pp. 286-287).

[22] Rugjer Bošković to Božo Bošković, 22 January 1754 (Truhelka, T-25, VIII,

43): ’Al fine delle mie Sezioni Coniche mi è convenuto aggiungere una dissertazione sulle trasformazioni de luoghi geometrici, e sull'infinito, la quale mi è tanto cresciuta sotto la penna, che è piu di un terzo del tomo.’

[23] ’Interea earum [curvarum] ductus hic definitus plurimum proderit ad quaedam infiniti mysteria evolvenda, et cognoscendam intimius continuitatis geometricae legem, ac ipsa plurimorum casuum contemplatio, et locorum generalis constructio sibi ubique respondens, ad Geometriae ipsius indolem, miram sanè, percipiendam pariter plurimum proderit.’ (Boscovich, 1754b, n. 692, p. 312.)

[24] ’Multa autem continet, quae licet scitu sane dignissima, ego quidem nusquam alibi offendi, multa, quae licet alibi etiam occurrant saepe, nusquam ego quidem ad certos reperi redacta canones, et geometrica methodo pertractata.’ (Boscovich, 1754c, p. XVIII.)

[25] ’Ea tamen pro novis venditare non audeo; cum mihi quidem inscitiae meae culpa, nova esse possint, licet fortasse sint apud Litterariam Remp[ublicam] vetustissima.’ (Boscovich, 1754c, p. XVIII.)

[26] ’…canones, qui per universam late Geometriam observantur, ...’ (Boscovich, 1754b, n. 759, p. 368).

[27] ’Porro singuli Canones demonstrantur accurate.’ (Boscovich, 1754c, p. XXIII.)

[28] Rugjer Bošković to Francesco Puccinelli, 15 November 1763, kept in the Archivum Historicum Societatis Iesu in Rome, Opp. Nostrorum 89, f. 2r: ’Ed ora non sono più in stato da farmene padrone.’ See also Marković’s comments on this letter in Marković (1968, pp. 62 and 506) and Marković (1969, p. 741).

[29] ’Sed ea omnia, et universam hanc theoriam nostram [= virium theoriam] multo diligentius excolemus, et propugnabimus brevi longiore opere, in quo et totam infinitesimorum theoriam a sola exclusione saltus pendere ostendemus, et praecipua mechanicae elementa proferemus nova methodo demonstrata.’ (Boscovich, 1748, n. 54, p. 23.)

[30] 'Occurrent autem identidem quaedam etiam infiniti mysteria, quae eo usque excrescent, ut infiniti extensi impossibilitatem demum suadent, ac ad indefinitorum, sive indefinite parva sint, sive indefinite magna, theoriam, quam alio opere pertractabimus, nos deducent.' (Boscovich, 1754b, n. 759, p. 368.)

[31] 'Consequetur aliud agens de infinitis, et infinite parvis, quae mihi indefinita sunt, quorum naturam explicabo, ordines diggeram, elementa tradam geometrico rigore demonstrata, et ex iis ad curvarum generales proprietates gradum faciam, cuspides, flexus contrarios, crura infinita, contactus, oscula, evolutas, maximorum, et minimorum theoriam, atque alia ejusmodi evolvam, ac singulares praecipuarum, et maxime utilium curvarum proprietates deducam, ac demonstrabo.' (Boscovich, 1754c, pp. XXV—XXVI.)

[32] See, e.g. the following attitudes: 'Quoniam tamen id ipsum admodum facile praestari potest ope calculi infinitesimalis admodum elementaris, …' (Boscovich, 1755c, n. 61, p. 408); '... subnormalis, quae ex formulis elementaribus calculi infinitesimalis, ..., debet esse —xdx/dy.' (Boscovich, 1755c, n. 62, p. 408.)

[33] 'Porro illud accuratissime per Geometriam demonstrari potest, ut quarto meorum Elementorum tomo demonstrabo, nullum esse arcum curvae cujusvis continuum, in quo non adsint infinita puncta circulum osculatorem habentia, ...' (Boscovich, 1755c, n. 259, p. 483.)

[34] 'Is quidem generaliter per calculum infinitesimalem proponit generalem solutionem problematis, quo data graduum serie inveniatur curva, ... Ejusmodi problematis generalis constructionem hic proponam solius Geometriae ope, ut superiora etiam pertractavi, ac ad meam de re tota sententiam, quam primo opusculo proposui post comparationes nonnullas demum delabar.' (Boscovich, 1755c, n. 306, pp. 502-503.)

[35] 'Nobis verò admittentibus in minimis punctorum distantiis repulsionem potius, quam attractionem, eamque positivo argumento probantibus ex

exclusione saltus, sive ex lege continuitatis, quam in omni motu, & mutatione quantitatum admittunt Recentiores passim, & quam inductio admodum ampla confirmat, tum reliqua omnia, tum illa ipsa tria principia Gravitatis, Cohaesionis, Fermentationis ab unico tantum principio profluunt; in quo majorem sanè successum habuisse videri possumus eo, quem ipse [Nevvtonus] maximè desiderandum censuit, ubi adjecit: *Ex phaenomenis naturae duo, vel tria derivare generalia motus principia, & deinde explicare quemadmodum proprietates, & actiones rerum corporearum omnium ex principiis istis consequantur; id verò magnus esset factus in Philosophia progressus; etiamsi principiorum istorum causae nondum essent cognitae.*' (Boscovich, 1748, n. 58, pp. 24-25, emphasis added by Bošković.) See also Boscovich (1763, p. XVI). For a more complete and detailed treatment of Bošković's systematic exploration of Newton's views exposed in *Query 31* see Marković (1961, pp. 130-132) and Martinović (1987a, pp. 84-92).

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